Quantum Mechanics

Prof. J. K. Baria
Professor of Physics
V P & R P T P Science College,
Vallabh Vidyanagar 388 120

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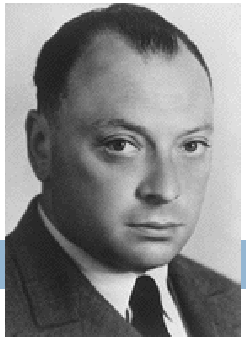
Spin Heroes

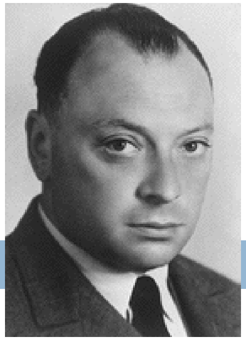
- Pauli's fourth quantum number
- Goudsmit and Uhlenbeck's proposal of spin encouraged by Ehrenfest
- Stern-Gerlach Experiment
- Zeeman Effect

Wolfgang Pauli



- Born 1900 in Vienna, died 1958 in Zurich
- Wolfgang Pauli's fourth quantum number
  - **Principle quantum number, n, size of the orbital in an atom**
  - **Angular quantum number, l, shape of the orbital in an atom**
  - **Magnetic quantum number, m, orientation in space of the orbital**
  - To distinguish between the two electrons in an orbital, we need a fourth quantum number!!!
- In January 1925 Pauli had proposed that the electron should be given an additional fourth quantum number which was a half integer

Wolfgang Pauli

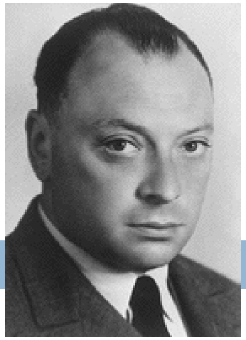


- This was one of the clues which led Uhlenbeck to arrive at the idea of electron spin. He wrote

  ... it occurred to me that , since (I had learned) each quantum number corresponds to a degree of freedom of the electron, Pauli's fourth quantum number must mean that the electron had an additional degree of freedom -- in other words the electron must be rotating.

- Pauli realized the importance of the extra angular momentum and postulated his exclusion principle (1945 Nobel Prize), which led to the quantum statistics of Fermi-Dirac distribution.

Wolfgang Pauli



- Pauli is infamous for a number of scathing remarks directed at his colleagues. Of one colleague's paper, he is purported to have said *"This isn't right. This isn't even wrong."*
- Pauli Effect: It was a standing joke among Wolfgang Pauli's colleagues that the famed theoretical physicist should be kept as far away from experimental equipment as humanly possible. His mere presence in a laboratory, it was said, would cause something to go wrong: the power would fail, vacuum tubes would suddenly leak, instruments would break or malfunction... Indeed, such was the frequency of Pauli-related incidents that the strange phenomenon came to be known as the 'Pauli Effect'.

Dutch Contribution

- Samuel Abraham Goudsmit and George Eugene Uhlenbeck, two graduate students of Ehrenfest (later assistant) at the University of Leiden in Netherlands

- In 1925 Uhlenbeck and Goudsmit postulated the existence of

  a new intrinsic property of particles that behaved like an angular momentum.

Dutch Contribution

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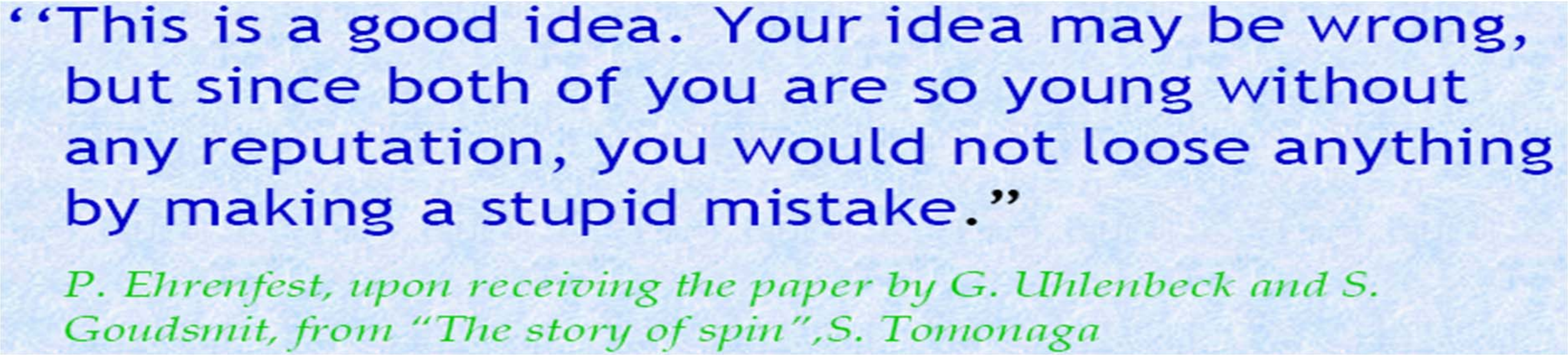
$$\vec{M} = \frac{g\mu_B}{\hbar}\vec{S}, \qquad \mu_B = \frac{e\hbar}{2m}$$

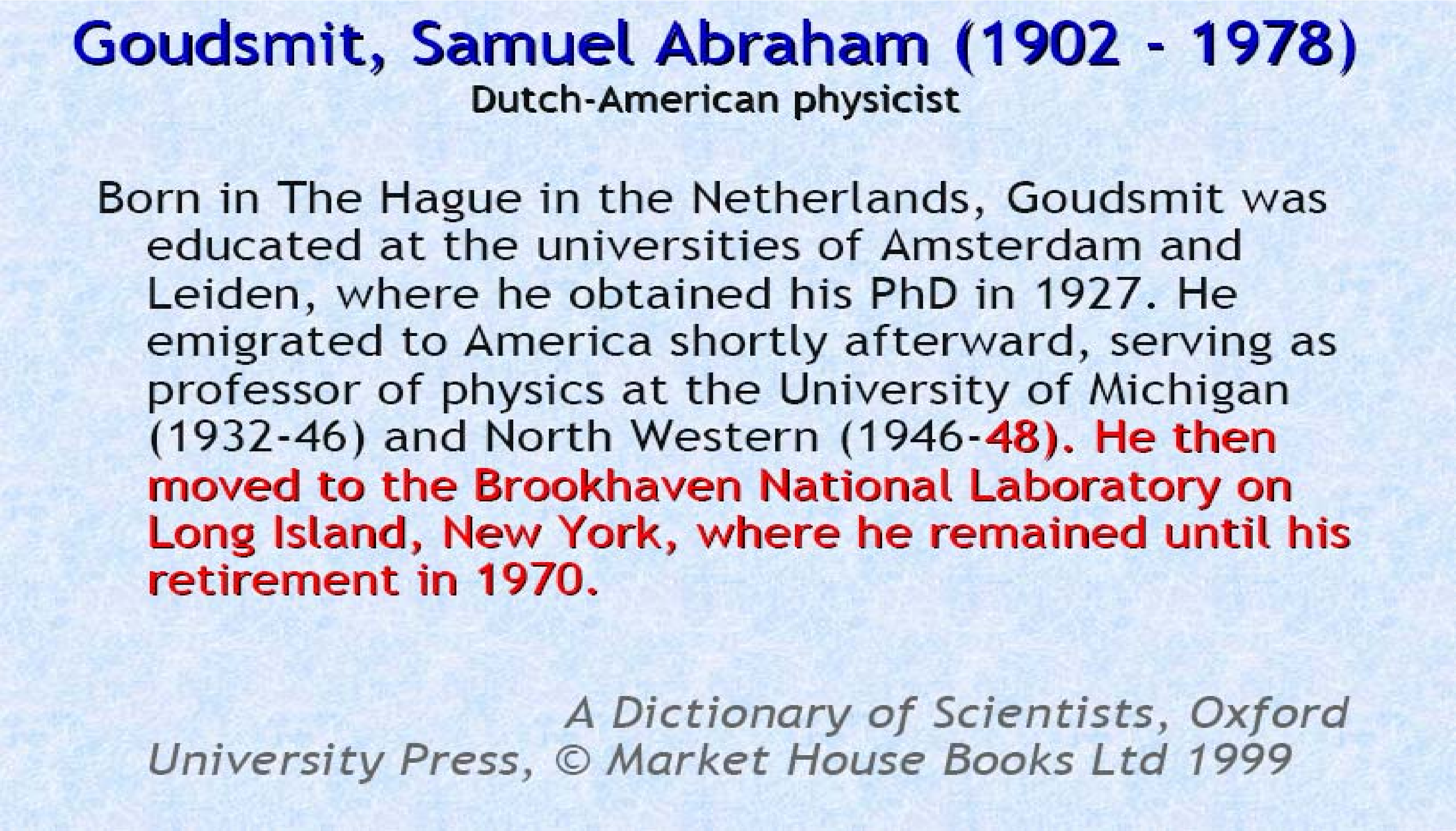
Dutch Contribution

- It is found that good fits to experimental data are obtained when *g*=2, which means that the *spin gyromagnetic ratio*, defined to be $g\mu_B/\hbar$ is twice as large as the *orbital gyromagnetic ratio* $\mu_B/\hbar$.
- Dirac later showed that spin arises very naturally in a correct relativistic formulation of the quantum theory. This formulation is embodied in the relativistic generalization of the Schrödinger equation called the Dirac equation.



‘‘This is a good idea. Your idea may be wrong, but since both of you are so young without any reputation, you would not loose anything by making a stupid mistake.’’

P. Ehrenfest, upon receiving the paper by G. Uhlenbeck and S. Goudsmit, from “The story of spin”,S. Tomonaga



Goudsmit, Samuel Abraham (1902 - 1978)

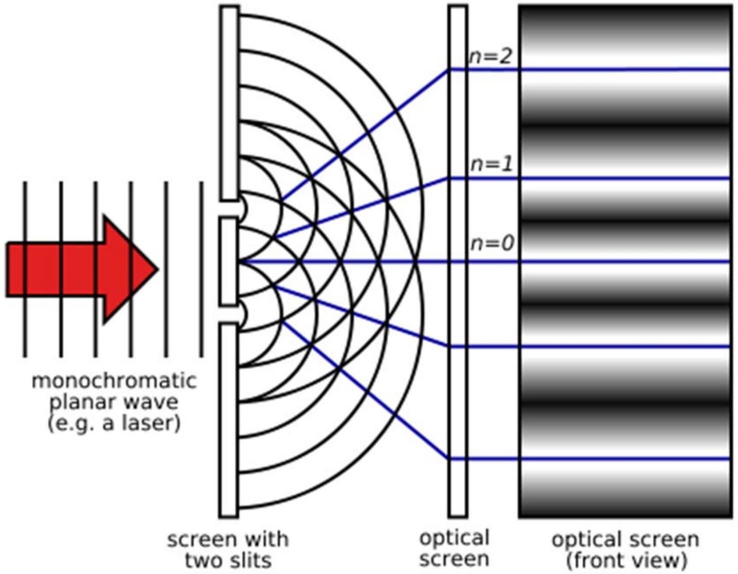
Dutch-American physicist

Born in The Hague in the Netherlands, Goudsmit was educated at the universities of Amsterdam and Leiden, where he obtained his PhD in 1927. He emigrated to America shortly afterward, serving as professor of physics at the University of Michigan (1932-46) and North Western (1946-**48). He then moved to the Brookhaven National Laboratory on Long Island, New York, where he remained until his retirement in 1970.**

A Dictionary of Scientists, Oxford University Press, © Market House Books Ltd 1999

Double-slit experiments:

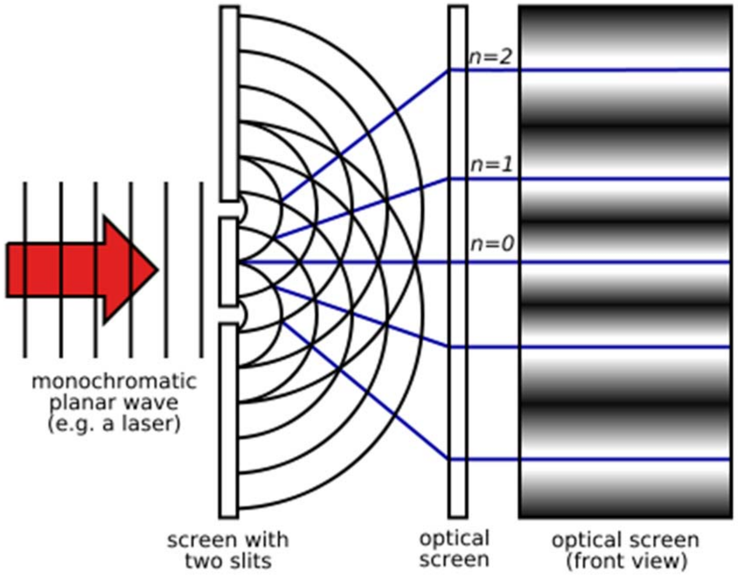
Light:

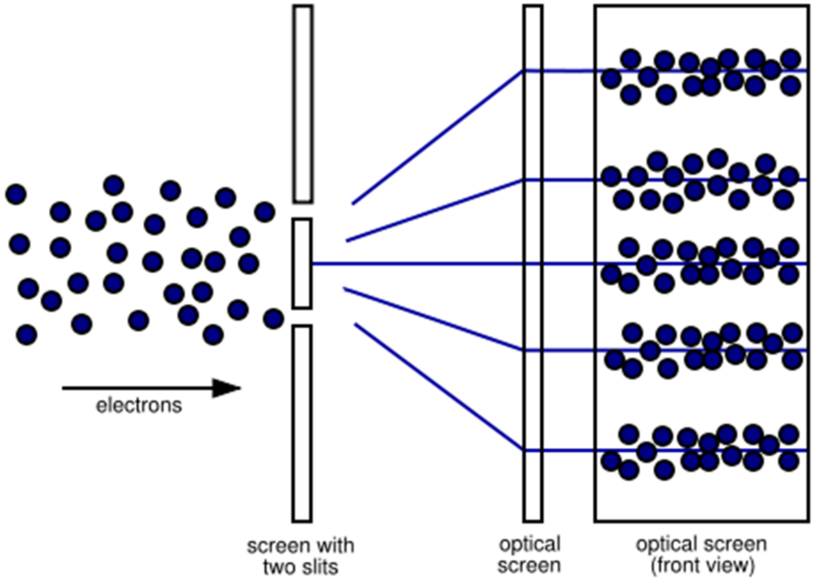


Double-slit experiments:

Light:

Electrons:





Individual electrons:

- In previous experiments many electrons were diffracted (or show interference)
- Will one get the same result for a single electron?
- Such experiments were performed
  - Intensity of the electron beam was so low that only one electron at a time proceeds
  - Still diffraction (and interference) patterns, and not diffused scattering, were observed, confirming that

Thus individual electrons possess wave properties!!!

Complimentarity Principle:

The particle and the wave models are COMPLIMENTARY

No measurements can simultaneously reveal the particle and the wave properties of matter

WAVE PARTICLE DUALITY

Evidence for wave-particle duality

- Photoelectric effect
- Compton effect

- Electron diffraction
- Interference of matter-waves

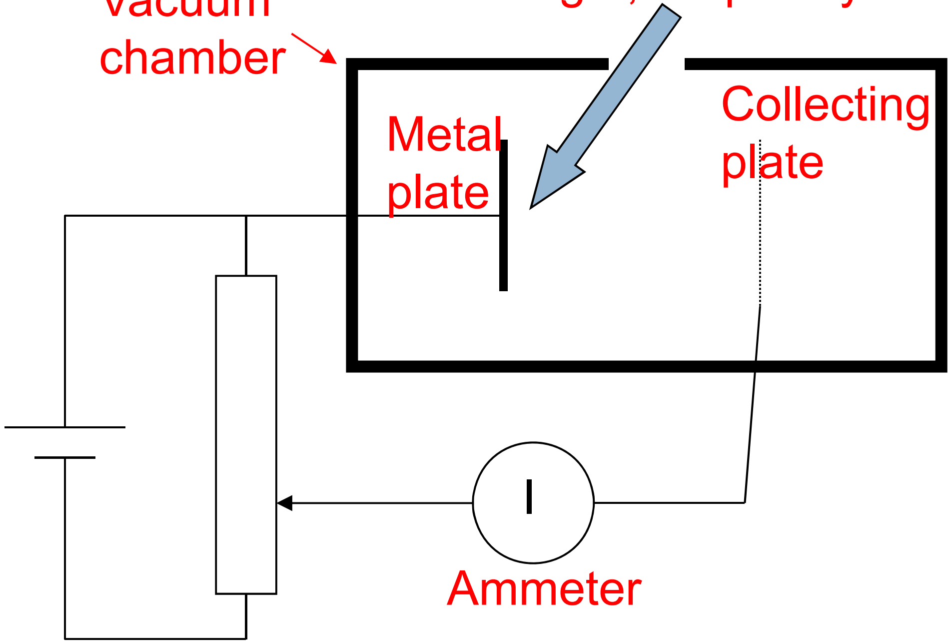
Consequence: Heisenberg uncertainty principle

PHOTOELECTRIC EFFECT

Hertz

J.J. Thomson

When UV light is shone on a metal plate in a vacuum, it emits charged particles (Hertz 1887), which were later shown to be electrons by J.J. Thomson (1899).



Vacuum chamber

Light, frequency v

Metal plate

Collecting plate

Ammeter

Potentiostat

Classical expectations

Electric field E of light exerts force $\mathbf{F}=-e\mathbf{E}$ on electrons. As intensity of light increases, force increases, so KE of ejected electrons should increase.

Electrons should be emitted whatever the frequency v of the light, so long as **E** is sufficiently large

For very low intensities, expect a time lag between light exposure and emission, while electrons absorb enough energy to escape from material

Photo electric effect (cont)

Actual results:

Maximum KE of ejected electrons is independent of intensity, but dependent on ν

For $\nu<\nu_0$ (i.e. for frequencies below a cut-off frequency) no electrons are emitted

There is no time lag. However, rate of ejection of electrons depends on light intensity.

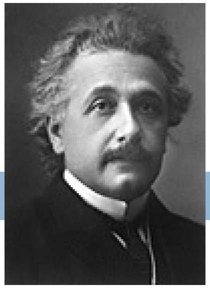
Einstein's interpretation (1905):

Light comes in packets of energy (*photons*)

$$E = h\nu$$

An electron absorbs a single photon to leave the material

Einstein



Millikan

The maximum KE of an emitted electron is then

$$K_{\max} = h\nu - W$$

Planck constant: universal constant of nature

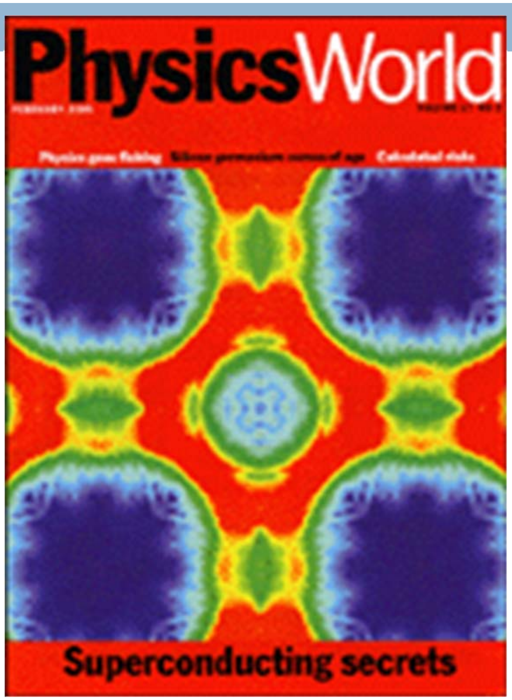
$$h = 6.63\times10^{-34}\,\text{Js}$$

Work function: minimum energy needed for electron to escape from metal (depends on material, but usually 2-5eV)

Verified in detail through subsequent experiments by Millikan

Photoemission experiments today

Modern successor to original photoelectric effect experiments is *ARPES (**A**ngle-**R**esolved **P**hoto**e**mission **S**pectroscopy)*



February 2000

Emitted electrons give information on distribution of electrons within a material as a function of energy *and* momentum

SUMMARY OF PHOTON PROPERTIES

Relation between particle and wave properties of light

Energy and frequency $E = h\nu$

Also have relation between momentum and wavelength

Relativistic formula relating energy and momentum $\boxed{E^2 = p^2c^2 + m^2c^4}$

For light $E = pc$ and $c = \lambda\nu$

$$\boxed{p = \frac{h}{\lambda} = \frac{h\nu}{c}}$$

Also commonly write these as

$$E = \hbar\omega \qquad p = \hbar k \qquad \omega = 2\pi\nu \qquad k = \frac{2\pi}{\lambda} \qquad \hbar = \frac{h}{2\pi}$$

wavevector

angular frequency

hbar

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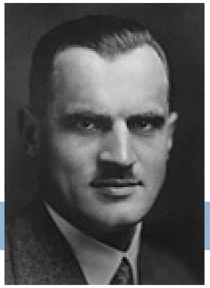
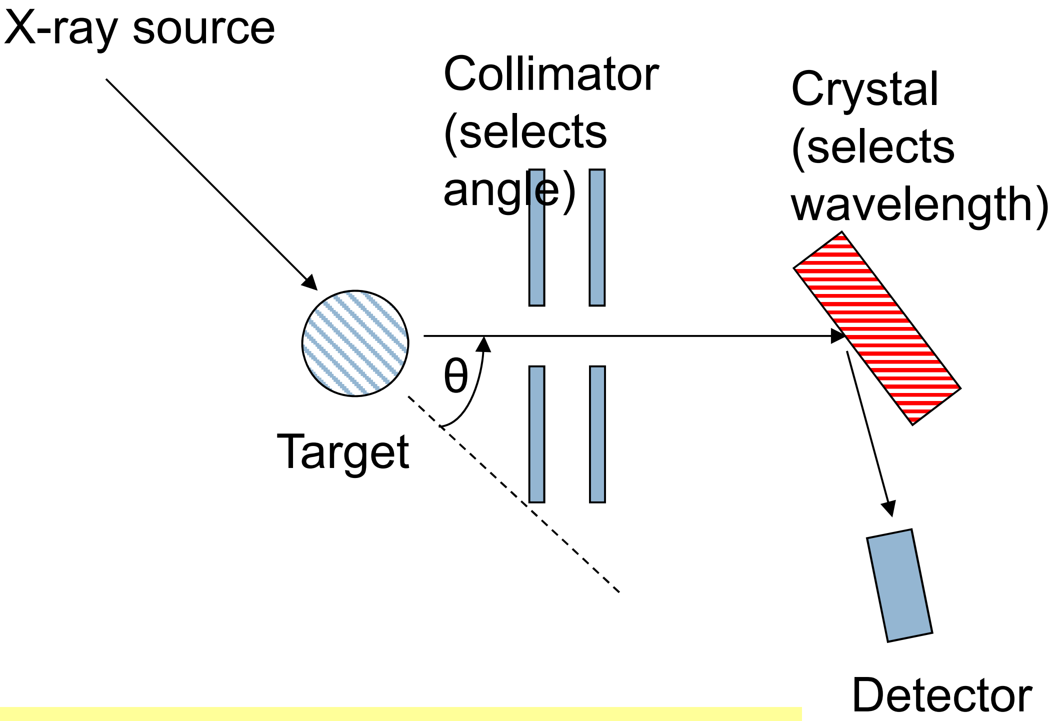
Compton



Compton (1923) measured intensity of scattered X-rays from solid target, as function of wavelength for different angles. He won the 1927 Nobel prize.



X-ray source

Collimator (selects angle)

Crystal (selects wavelength)

θ

Target

Detector

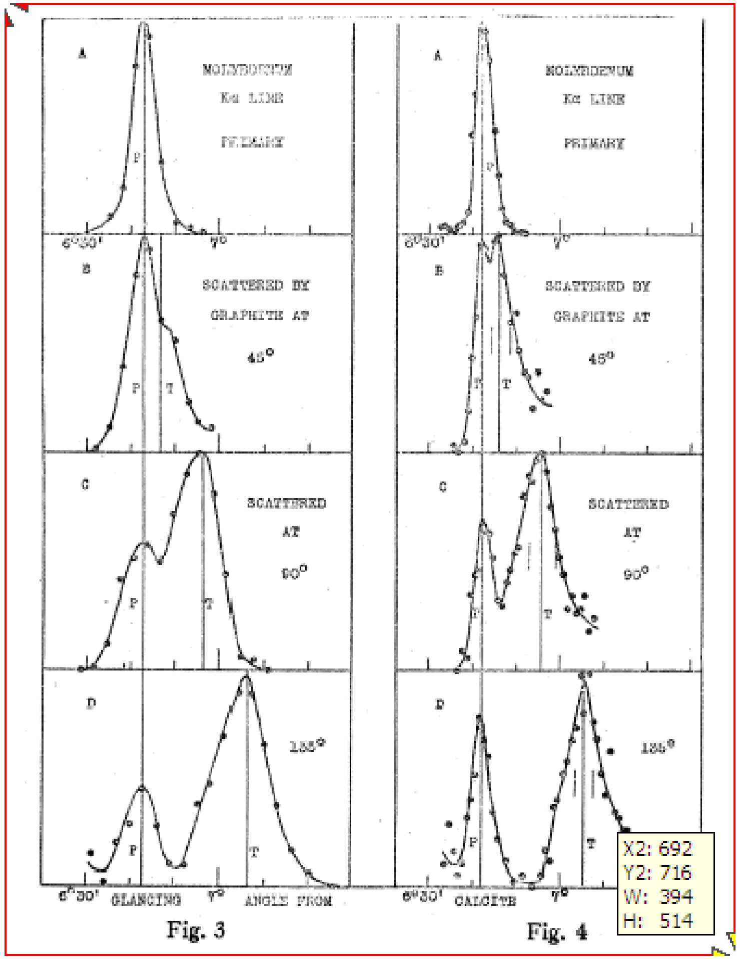


Fig. 3

Fig. 4

Result: peak in scattered radiation shifts to longer wavelength than source. Amount depends on *θ* (but not on the target material).

A.H. Compton, *Phys. Rev.* **22** 409 (1923)

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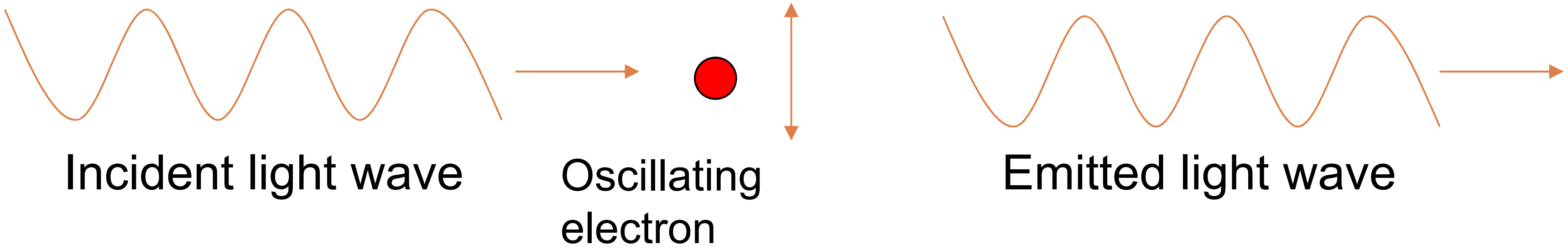
Classical picture: oscillating electromagnetic field causes oscillations in positions of charged particles, which re-radiate in all directions at *same frequency and wavelength* as incident radiation.

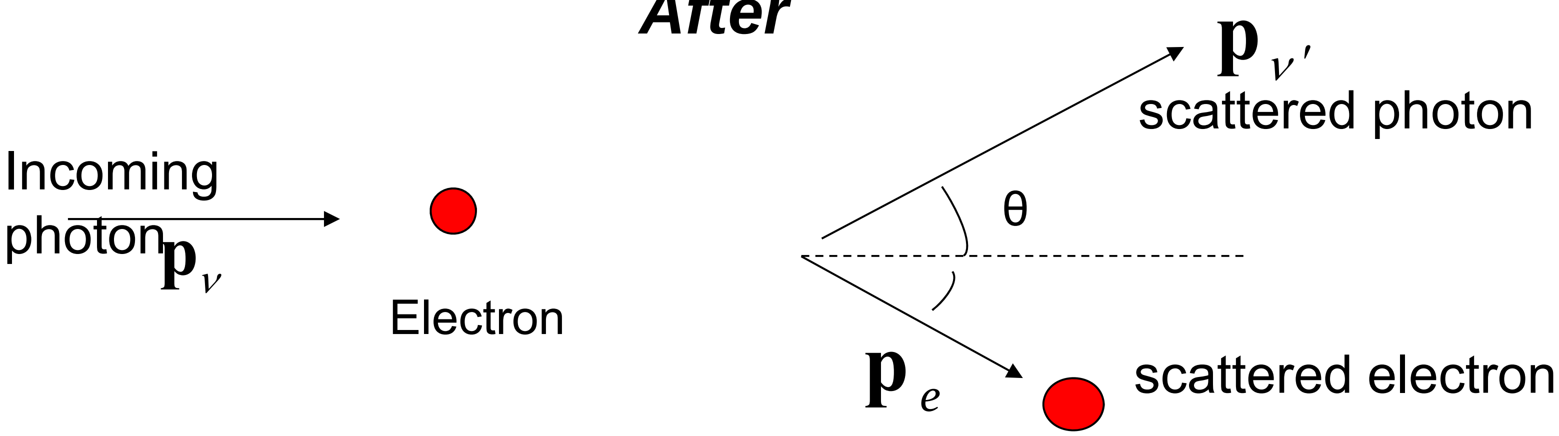
Change in wavelength of scattered light is completely unexpected classically



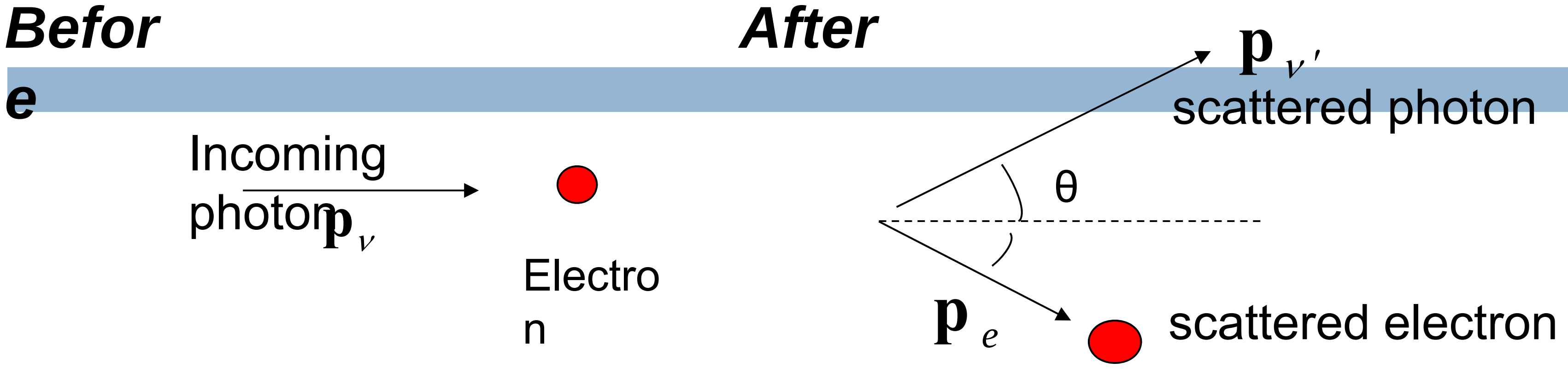
Compton's explanation: "billiard ball" collisions between particles of light (X-ray photons) and electrons in the material

Before

After



COMPTON SCATTERING (cont)



Conservation of energy

$$h\nu + m_e c^2 = h\nu' + \left(p_e^2 c^2 + m_e^2 c^4\right)^{1/2}$$

Conservation of momentum

$$\mathbf{p}_\nu = \frac{h}{\lambda}\hat{\mathbf{i}} = \mathbf{p}_{\nu'} + \mathbf{p}_e$$

From this Compton derived the change in wavelength

$$\boxed{\begin{aligned}\lambda' - \lambda &= \frac{h}{m_e c}\left(1-\cos\theta\right)\\ &= \lambda_c\left(1-\cos\theta\right) \geq 0\end{aligned}}$$

$$\lambda_c = \text{ Compton wavelength } = \frac{h}{m_e c} = 2.4\times10^{-12}\,\text{m}$$

COMPTON SCATTERING (cont)

Note that, at all angles
there is also an unshifted peak.

This comes from a collision
between the X-ray photon and
the nucleus of the atom

$$\boxed{\lambda' - \lambda = \frac{h}{m_N c}(1-\cos\theta) \approx 0}$$

since $m_N \gg m_e$

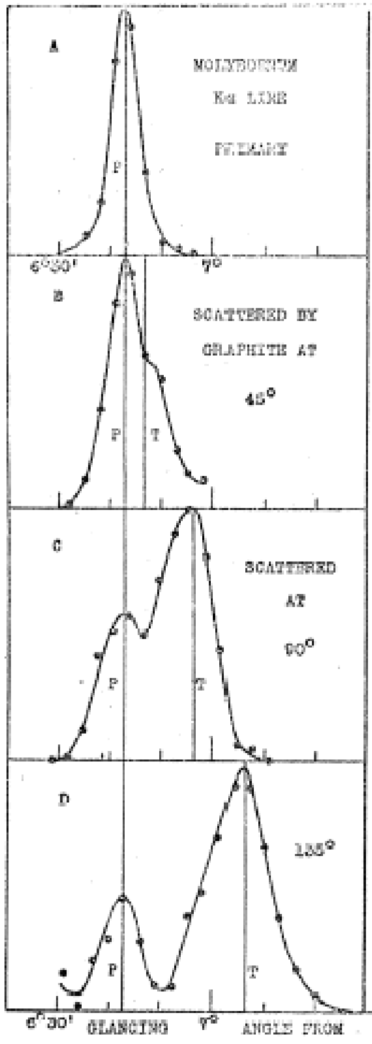


Fig. 3

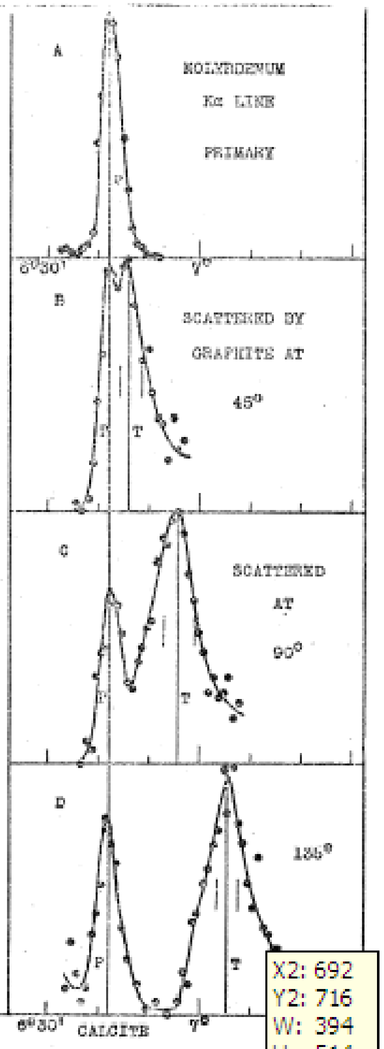


Fig. 4

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- gh#Eurjolh#srvwxodwhg#wkdw#wkh# z dyhohqjwk#ri#p dwwhu# lv#uhodwhg#wr#p rphqwxp #dv

$$\lambda = \frac{h}{p}$$

- Wklv#lv#fdoohg# wkh#gh#Eurjolh#z dyhohqjwk1

Nobel prize, 1929

Z k|#k 2#s B#Z runv#iru#skrwrqv

- ☐ Z h#dujxh#wkdw## $\lambda = \frac{h}{p}$ #dssolhv#wr#hyhu|wklqj
- ☐ Skrwrqv#dqg#irrwedoov#
  erwk#iroorz#wkh#vdph#uhodwlrq1#
- ☐ Hyhu|wklqj#kdv#erwk#
  z dyh0olnh dqg#sduwlfoh0olnh#surshuwlhv

Wavelength of massive object

$$\left.\begin{array}{l}\text{☐ de Broglie wavelength = } \lambda = \dfrac{h}{p} \\ \\ \bullet\ \ p{=}mv \text{ for a nonrelativistic } (v{<<}c) \text{ particle } \textbf{with mass}. \end{array}\right\} \quad \lambda = \frac{h}{mv}$$

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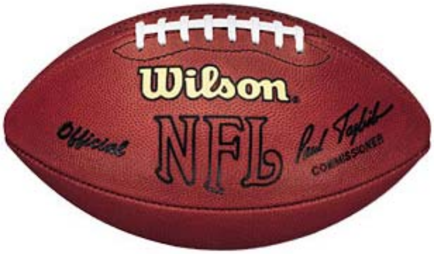

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  +5: 065 p 2v,

- Momentum: $$mv = (0.4\ \ kg)(30\ \ m/s) = 12\ \ kg - m/s$$

$$\lambda = \frac{h}{p} = \frac{6.6 \times 10^{-34}\ \ J - s}{12\ \ kg - m/s} = 5.5 \times 10^{-35} m = \boxed{5.5 \times 10^{-26} nm}$$

This is very small

- 1 nm = 10^{-9} m
- Wavelength of red light = 700 nm
- Spacing between atoms in solid ~ 0.25 nm
- Wavelength of football = 10^{-26} nm

- What makes football wavelength so small?

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

Large mass, large momentum
short wavelength

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    - Hqhuj|#ohyhov#duh#txdqwl}hg/#exw#glvfuhwhqhvv#lv#wrr#vpdoo#wr#eh#ghwhfwhg1
    - Zdyh#surshuwlhv#dovr#wrr#vpdoo#wr#eh#ghwhfwhg

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- Qhhg#d#pdwhuldo#remhfw#wr#vkrz#zdyh#hiihfw
- Hohfwurq#lv#d#yhu|#oljkw#sduwlfoh
- Pdvv#ri#hohfwurq#@#<14{43^{064}#nj

$$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{6 \times 10^{-34}\, J - s}{\left(9 \times 10^{-31}\, kg\right) \times \left(velocity\right)}$$

Wavelength depends on mass and velocity

Larger velocity, shorter wavelength

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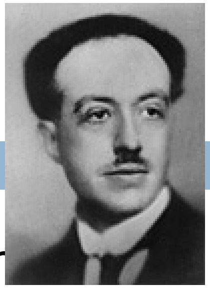
In 1924 Einstein wrote:- “ There are therefore now two theories of light, both indispensable, and … without any logical connection.”

Evidence for wave-nature of light
- Diffraction and interference

Evidence for particle-nature of light
- Photoelectric effect
- Compton effect

- Light exhibits diffraction and interference phenomena that are *only* explicable in terms of wave properties
- Light is always detected as packets (photons); if we look, we never observe half a photon
- Number of photons proportional to energy density (i.e. to square of electromagnetic field strength)

De Broglie



MATTER WAVES

We have seen that light comes in discrete units (photons) with particle properties (energy and momentum) that are related to th wave-like properties of frequency and wavelength.

In 1923 Prince Louis de Broglie postulated that ordinary matter can have wave-like properties, with the wavelength λ related to momentum p in the same way as for light

de Broglie relation

$$\boxed{\lambda = \frac{h}{p}}$$

← Planck's constant

$$h = 6.63 \times 10^{-34}\ \mathrm{Js}$$

de Broglie wavelength

NB wavelength depends on momentum, not on the physical size of the part

Prediction: We should see diffraction and interference of matter w

Estimate some de Broglie wavelengths

- Wavelength of electron with 50eV kinetic energy

$$K = \frac{p^2}{2m_e} = \frac{h^2}{2m_e\lambda^2} \Rightarrow \lambda = \frac{h}{\sqrt{2m_eK}} = 1.7\times10^{-10}\,\mathrm{m}$$

- Wavelength of Nitrogen molecule at room temperature

$$K = \frac{3kT}{2}, \quad \mathrm{Mass} = 28\mathrm{m_u}$$

$$\lambda = \frac{h}{\sqrt{3MkT}} = 2.8\times10^{-11}\,\mathrm{m}$$

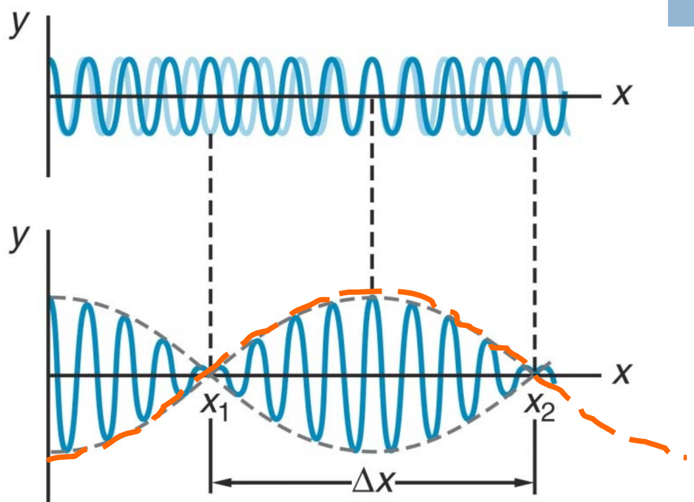
- Wavelength of Rubidium(87) atom at 50nK

$$\lambda = \frac{h}{\sqrt{3MkT}} = 1.2\times10^{-6}\,\mathrm{m}$$

Skdvh#dqg#Jurxs#Yhorflwlhv

This ***phase*** velocity $v_p = \frac{\omega}{k} = \frac{E}{p} = \frac{\gamma mc^2}{\gamma mv} = \frac{c^2}{v} > c$

- no limitations on the phase velocity, (phase of a plane wave does not carry any information)



$$\leftarrow \text{"}\lambda\text{"} \approx \frac{2\pi}{\Delta k} \rightarrow$$

The observable is the ***group*** velocity (the velocity of propagation of a wave "packet" or wave "group". Let's consider the superposition of two harmonic waves with slightly different frequencies (ω>>Δω, k>>Δk):

$$y_1 = A\cos(\omega t - kx)$$

$$y_2 = A\cos\left[(\omega + \Delta\omega)t - (k + \Delta k)x\right]$$

$$\cos\alpha + \cos\beta = 2\cos\left[\frac{1}{2}(\alpha+\beta)\right]\cos\left[\frac{1}{2}(\alpha-\beta)\right]$$

$$y = y_1 + y_2 = 2A\cos\left[\frac{1}{2}\left\{(2\omega+\Delta\omega)t - (2k+\Delta k)x\right\}\right]\cos\left[\frac{1}{2}(\Delta\omega\cdot t - \Delta k\cdot x)\right]$$

$$\approx \underbrace{2A\cos(\omega t - kx)}_{\text{fast oscillations within the wave group}}\underbrace{\cos\left(\frac{\Delta\omega}{2}t - \frac{\Delta k}{2}x\right)}_{\text{"envelope"= wave group}}$$

The velocity of propagation of the wave packet:

$$\boxed{v_g = \frac{d\omega}{dk}}$$

-the ***group*** velocity

Jurxs#Yhorflw|#ri#gh#Eurjoh#z dyhv

$$v_g = \frac{d\omega}{dk} = \frac{dE}{dp} \qquad E = \sqrt{(pc)^2 + (mc^2)^2}$$

$$\boxed{v_g = v}$$

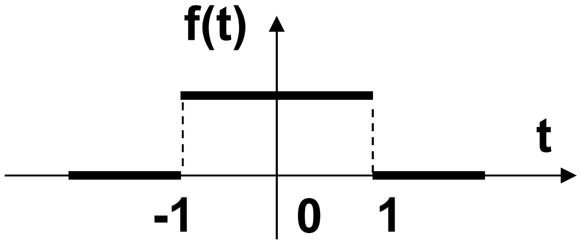
- the group velocity of de Broglie waves coincide with the particle's velocity

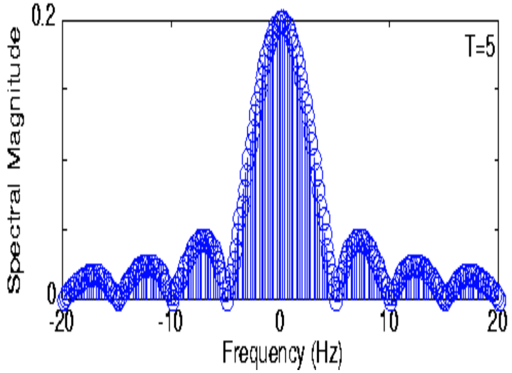
$$\frac{dE}{dp} = \frac{1}{2}\frac{2pc^2}{\sqrt{(pc)^2 + (mc^2)^2}} = \frac{pc^2}{E} = \frac{\gamma mv \cdot c^2}{\gamma mc^2} = v$$

$$\boxed{v_g v_p = c^2}$$

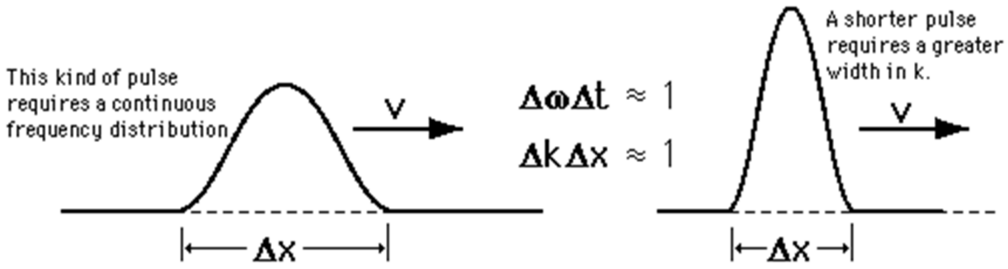
Periodic processes: discrete spectrum (Fourier series).

Aperiodic processes: continuous spectrum (represented as Fourier integral)





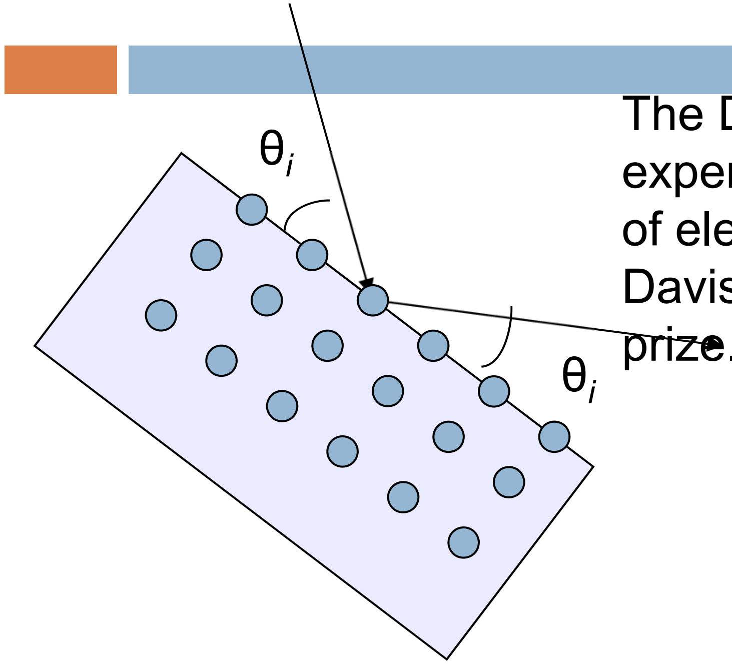
$$\mathrm{sinc}(\omega) = \begin{cases} 1, & \omega = 0 \\ \dfrac{\sin\omega}{\omega}, & \omega \neq 0 \end{cases}$$



$$\Delta\omega\Delta t \approx 1$$

$$\Delta k \Delta x \approx 1$$

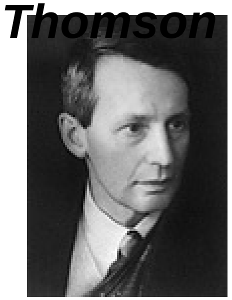
ELECTRON DIFFRACTION
The Davisson-Germer experiment (1927)



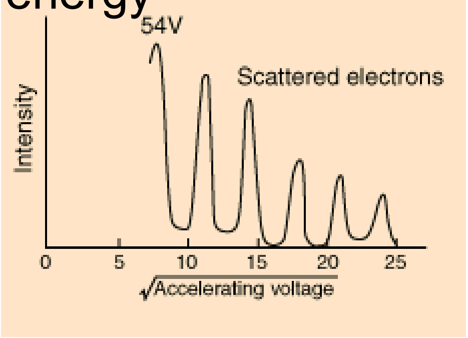
The Davisson-Germer experiment: scattering a beam of electrons from a Ni crystal. Davisson got the 1937 Nobel prize.

Davisson

G.P. Thomson

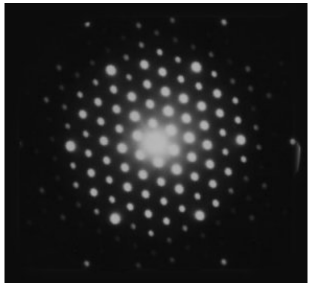


At fixed *angle*, find sharp peaks in intensity as a function of electron energy



Davisson, C. J., "Are Electrons Waves?," Franklin Institute Journal **205**, 597 (1928)

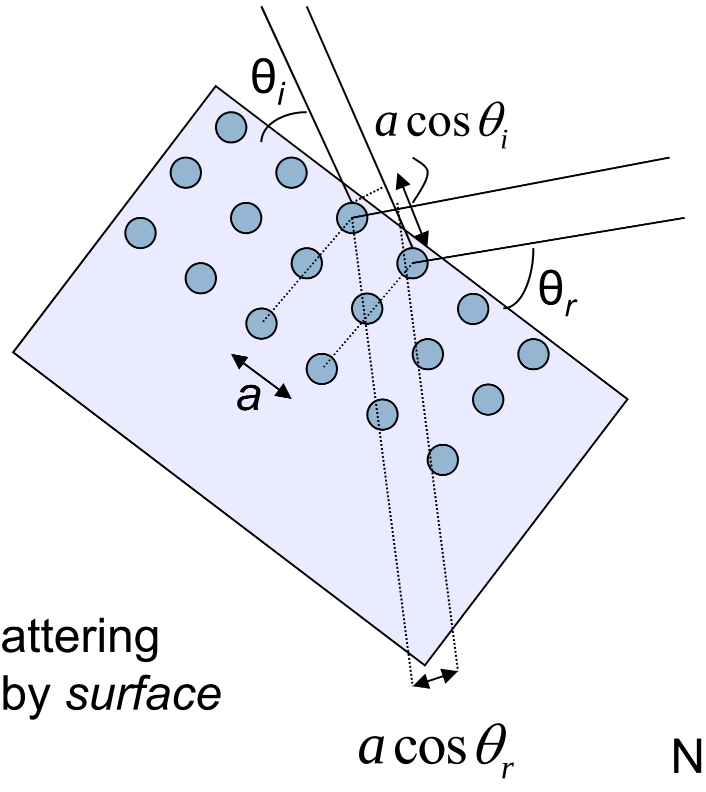
At fixed accelerating voltage (fixed electron energy) find a pattern of sharp reflected beams from the crystal



G.P. Thomson performed similar interference experiments with thin-film samples

ELECTRON DIFFRACTION (cont)

Interpretation: similar to Bragg scattering of X-rays from crystals



Path difference:

$$a(\cos\theta_r - \cos\theta_i)$$

Constructive interference when

$$a(\cos\theta_r - \cos\theta_i) = n\lambda$$

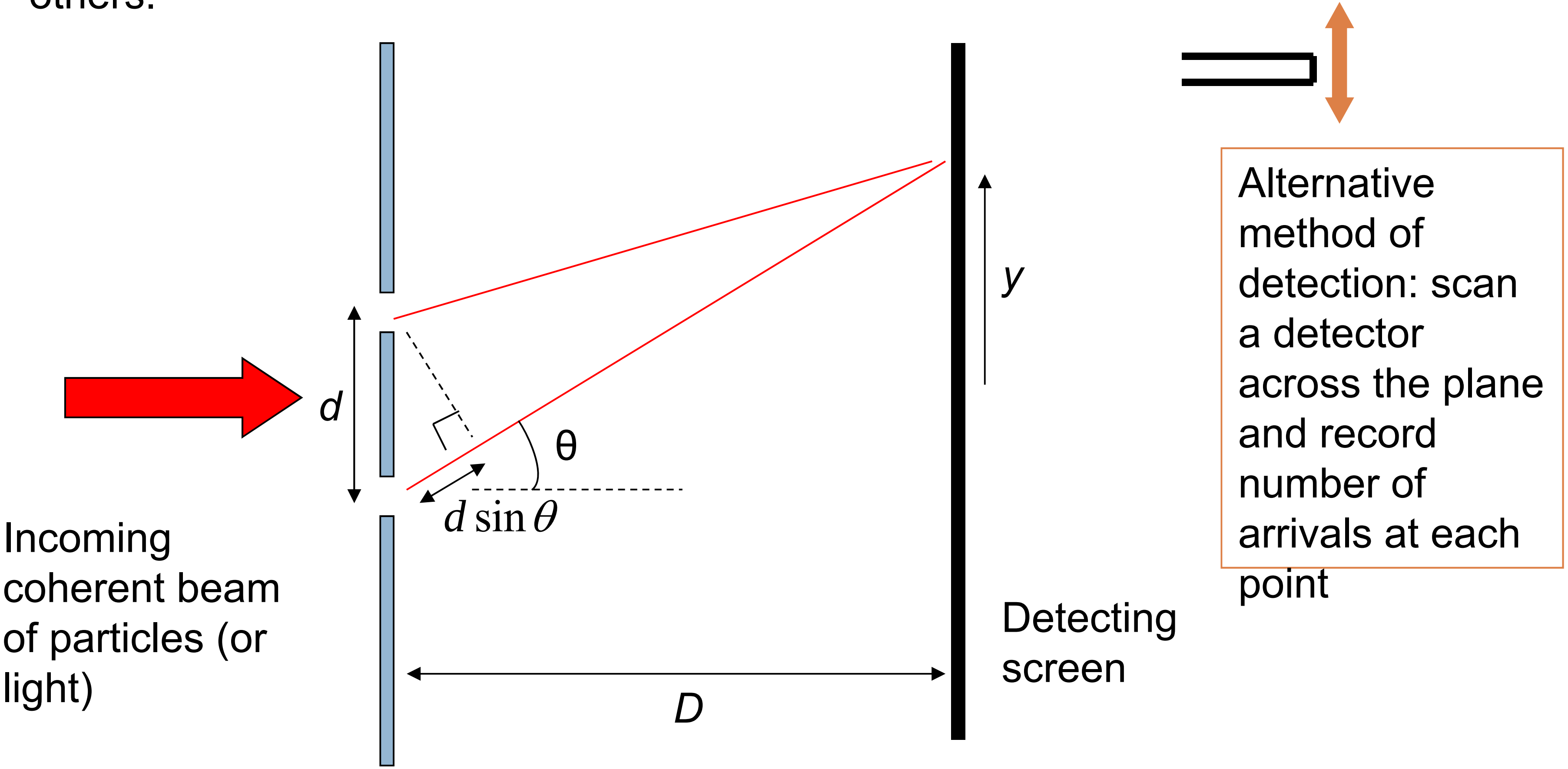
Electron scattering dominated by *surface* layers

Note θ_i and θ_r not necessarily equal

Note difference from usual “Bragg’s Law” geometry: the identical scattering planes are oriented *perpendicular* to the surface

THE DOUBLE-SLIT EXPERIMENT

Originally performed by Young (1801) to demonstrate the wave-nature of light. Has now been done with electrons, neutrons, He atoms among others.



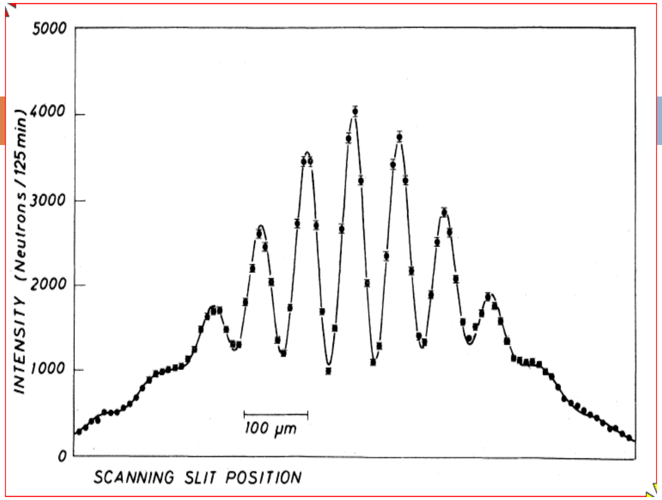
Incoming coherent beam of particles (or light)

Detecting screen

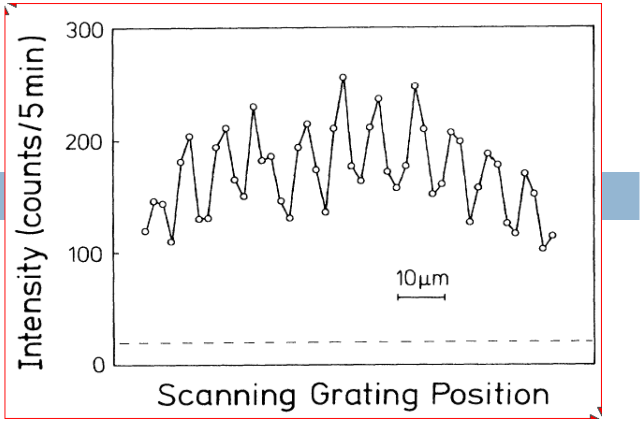
Alternative method of detection: scan a detector across the plane and record number of arrivals at each point

For particles we expect two peaks, for waves an interference pattern

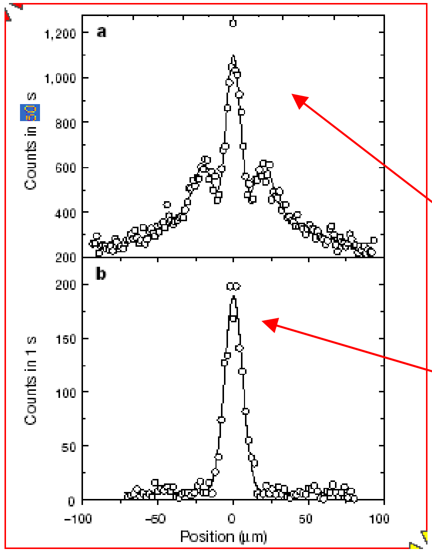
EXPERIMENTAL RESULTS



Neutrons, A Zeilinger *et al.* 1988 *Reviews of Modern Physics* **60** 1067-1073



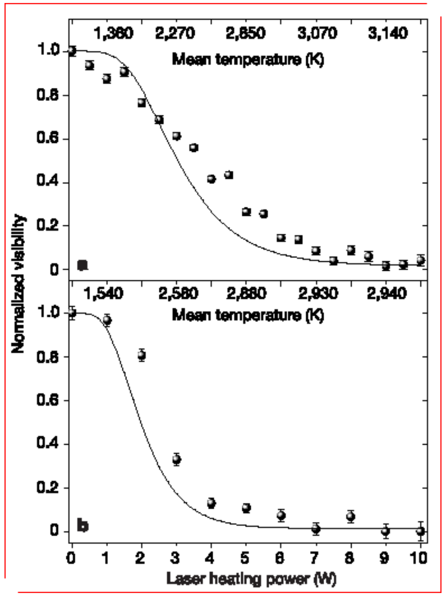
He atoms: O Carnal and J Mlynek 1991 *Physical Review Letters* **66** 2689-2692



C_{60} molecules: M Arndt *et al.* 1999 *Nature* **401** 680-682

With multiple-slit grating

Without grating



Fringe visibility decreases as molecules are heated. L. Hackermüller *et al.* 2004 *Nature* **427** 711-714

Interference patterns can not be explained classically - clear demonstration of

DOUBLE-SLIT EXPERIMENT WITH HELIUM ATOMS

(Carnal & Mlynek, 1991,Phys.Rev.Lett.,66,p2689)

Path difference: $d\sin\theta$

Constructive interference: $d\sin\theta = n\lambda$

Separation between maxima: $\Delta y = \dfrac{\lambda D}{d}$
(proof following)

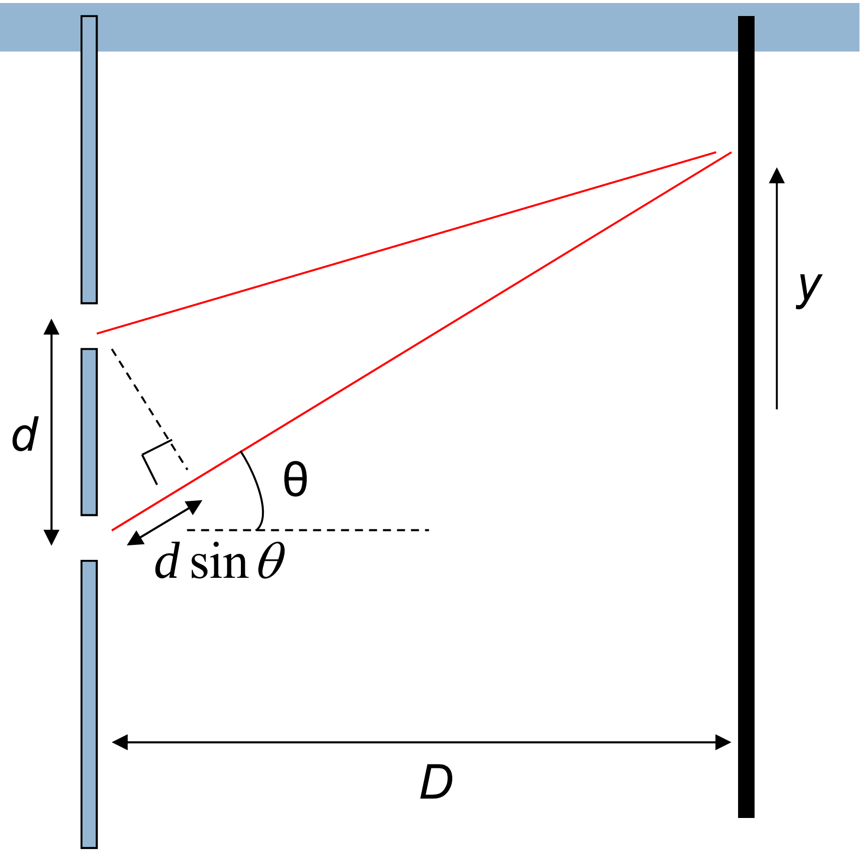
Experiment: He atoms at 83K,
with *d*=8μm and *D*=64cm

Measured separation: $\Delta y = 8.2\mu m$

Predicted de Broglie wavelength:

$$K = \frac{3kT}{2}, \quad \mathrm{Mass} = 4\mathrm{m_u}$$

$$\lambda = \frac{h}{\sqrt{3MkT}} = 1.03\times10^{-10}\,\mathrm{m}$$



Predicted separation: $\Delta y = 8.4 \pm 0.8\mu m$

Good agreement with experiment

FRINGE SPACING IN DOUBLE-SLIT EXPERIMENT

Maxima when: $d\sin\theta = n\lambda$

$D \gg d$ so use small angle approximation

$$\theta \approx \frac{n\lambda}{d}$$

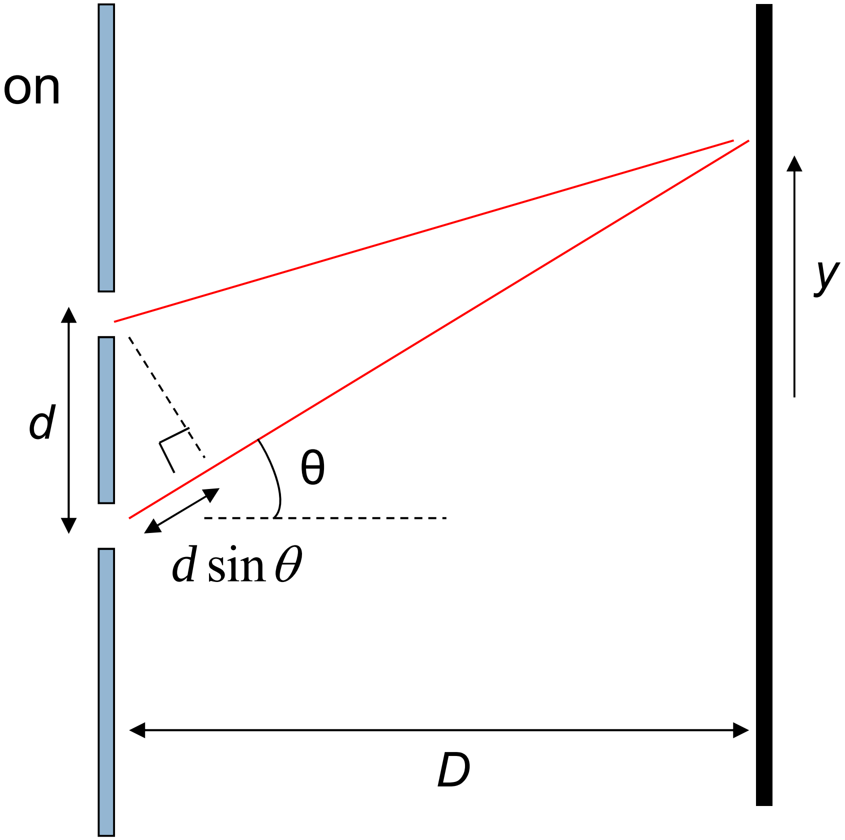
$$\Rightarrow \Delta\theta \approx \frac{\lambda}{d}$$

Position on screen $y = D\tan\theta \approx D\theta$

So separation between adjacent maxima:

$$\Delta y \approx D\Delta\theta$$

$$\Rightarrow \Delta y = \frac{\lambda D}{d}$$



DOUBLE-SLIT EXPERIMENT
INTERPRETATION

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The importance of the two-slit experiment has been memorably summarized by Richard Feynman: "…a phenomenon which is impossible, *absolutely* impossible, to explain in any classical way, and which has in it the heart of quantum mechanics. In reality it contains the *only* mystery."

DOUBLE-SLIT EXPERIMENT
BIBLIOGRAPHY

Some key papers in the development of the double-slit experiment during the 20th cen

- Performed with a light source so faint that only one photon exists in the apparatus at any one tim
  G I Taylor 1909 *Proceedings of the Cambridge Philosophical Society* **15** 114-115
- Performed with electrons
  C Jönsson 1961 *Zeitschrift für Physik* **161** 454-474,
  (translated 1974 *American Journal of Physics* **42** 4-11)
- Performed with single electrons
  A Tonomura *et al.* 1989 *American Journal of Physics* **57** 117-120
- Performed with neutrons
  A Zeilinger *et al.* 1988 *Reviews of Modern Physics* **60** 1067-1073
- Performed with He atoms
  O Carnal and J Mlynek 1991 *Physical Review Letters* **66** 2689-2692
- Performed with C60 molecules
  M Arndt *et al.* 1999 *Nature* **401** 680-682
- Performed with C70 molecules showing reduction in fringe visibility as temperature rises
  and the molecules “give away” their position by emitting photons
  L. Hackermüller et al 2004 *Nature* **427** 711-714
- Performed with Na Bose-Einstein Condensates
  M R Andrews *et al.* 1997 *Science* **275** 637-641

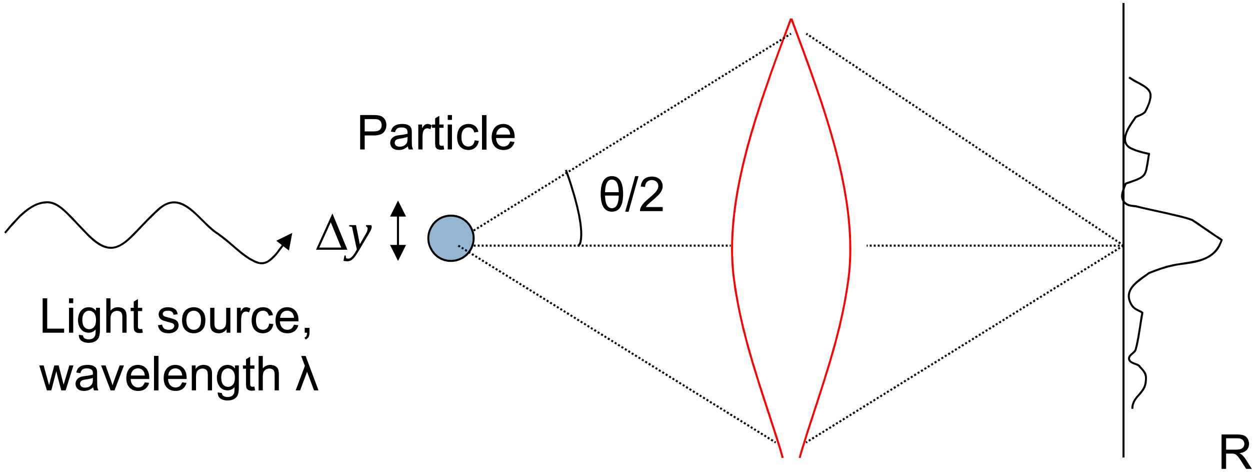
An excellent summary is available in *Physics World* (September 2002 issue, page 15)
and at http://physicsweb.org/ (readers voted the double-slit experiment “the most beautiful in phys

HEISENBERG MICROSCOPE AND THE UNCERTAINTY PRINCIPLE

(also called the Bohr microscope, but the thought experiment is mainly due to Heisenberg).

The microscope is an imaginary device to measure the position (y) and momentum (p) of a particle.

Heisenberg



Light source, wavelength λ

Lens, with angular diameter θ

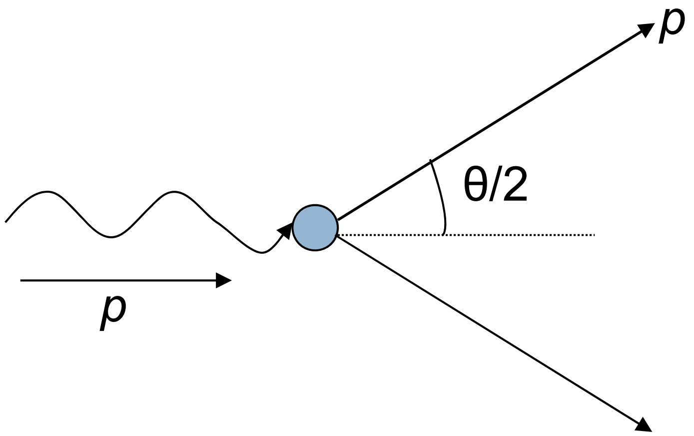
Resolving power of lens:

$$\Delta y \geq \frac{\lambda}{\theta}$$

HEISENBERG MICROSCOPE (cont)

Photons transfer momentum to the particle when they scatter.

Magnitude of p is the same before and after the collision. Why?



Uncertainty in *photon* y-momentum
= Uncertainty in *particle* y-momentum

$$\boxed{-p\sin(\theta/2) \le p_y \le p\sin(\theta/2)}$$

Small angle approximation

$$\Delta p_y = 2p\sin(\theta/2) \approx p\theta$$

de Broglie relation gives $p = h/\lambda$ and so $\Delta p_y \approx \dfrac{h\theta}{\lambda}$

From before $\Delta y \ge \dfrac{\lambda}{\theta}$ hence $\boxed{\Delta p_y \Delta y \approx h}$

HEISENBERG UNCERTAINTY PRINCIPLE.

Point for discussion

The thought experiment seems to imply that, while prior to experiment we have well defined values, it is the act of measurement which introduces the uncertainty by disturbing the particle's position and momentum.

Nowadays it is more widely accepted that quantum uncertainty (lack of determinism) is intrinsic to the theory.

HEISENBERG UNCERTAINTY PRINCIPLE

We will show formally (section 4)

$$\boxed{\begin{aligned} \Delta x \Delta p_x &\geq \hbar / 2 \\ \Delta y \Delta p_y &\geq \hbar / 2 \\ \Delta z \Delta p_z &\geq \hbar / 2 \end{aligned}}$$

HEISENBERG UNCERTAINTY PRINCIPLE.

We cannot have simultaneous knowledge of ‘conjugate’ variables such as position and momenta.

Note, however, $\boxed{\Delta x \Delta p_y \geq 0}$ etc

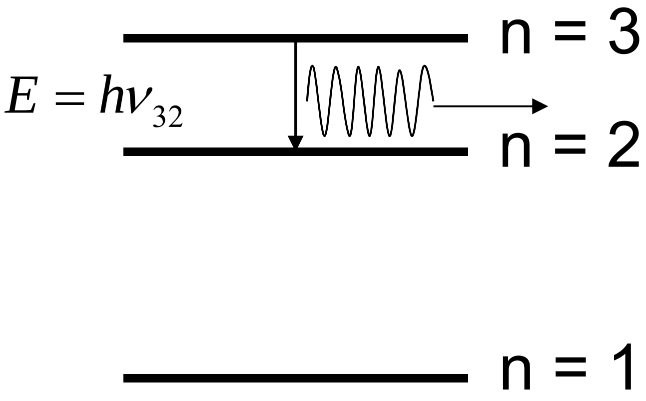
Arbitary precision is possible in principle for position in one direction and momentum in another

HEISENBERG UNCERTAINTY PRINCIPLE

There is also an energy-time uncertainty relation

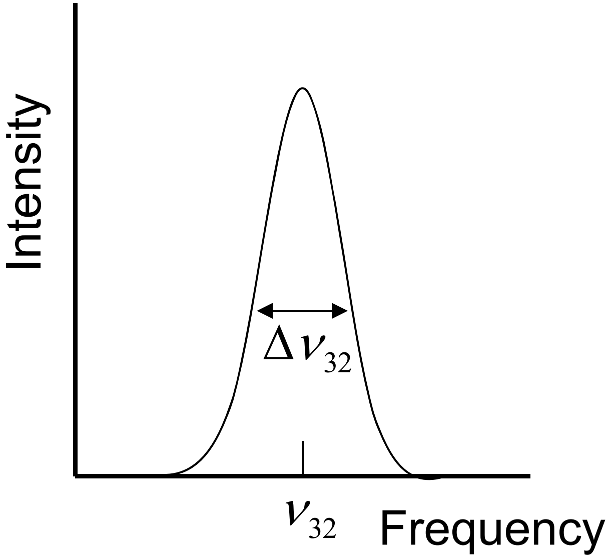
$$\boxed{\Delta E \Delta t \geq \hbar / 2}$$

Transitions between energy levels of atoms are not perfectly sharp in frequency.



An electron in *n* = 3 will spontaneously decay to a lower level after a lifetime of order $t \sim 10^{-8}\,\mathrm{s}$

There is a corresponding 'spread' in the emitted frequency



CONCLUSIONS

Light and matter exhibit wave-particle duality

$$E = h\nu \qquad p = \frac{h}{\lambda}$$

Relation between wave and particle properties
given by the de Broglie relations,

Evidence for particle properties of light
Photoelectric effect, Compton scattering

Evidence for wave properties of matter
Electron diffraction, interference of matter waves (electrons, neutrons, He atoms, C60 molecules)

$$\Delta x \Delta p_x \geq \hbar / 2$$
$$\Delta y \Delta p_y \geq \hbar / 2$$
$$\Delta z \Delta p_z \geq \hbar / 2$$

Heisenberg uncertainty principle limits simultaneous knowledge of conjugate variables

George Eugene Uhlenbeck

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George Eugene Uhlenbeck

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Introduction to Quantum mechanics

Some History

Early 20th century: Some revolutionary ideas from bright minds…

Werner **Heisenberg**
1901-1976

Uncertainty Principle

Erwin **Schrödinger**
1887-1961

Schrödinger Equation

Wolfgang **Pauli**
1900- 1958

Pauli exclusion principle

Introduction to Quantum mechanics

Essential ideas

1) Uncertainty principle:
Conjugates quantities of a particle (ex: position & momentum)
can not be known simultaneously within a certain accuracy limit

2) Quantization:
The measurement of a physical quantity in a confined system results in quanta
(the measured values are discrete)

3) Wave-particle duality:
All particles can be described as waves (travelling both in space and in time)
The state of the particle is given by a wave function $\Psi(x,t)$

4) Extrapolation to classical mechanics:
The laws of classical Newtonian mechanics are the extrapolation of the
laws of quantum mechanics for large systems with very large number of particles

Introduction to Quantum mechanics

Essential ideas

Schrödinger equation (1926)

$$i\hbar\frac{\partial\Psi}{\partial t}=-\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2}+V\Psi$$

Erwin **Schrödinger**
1887-1961

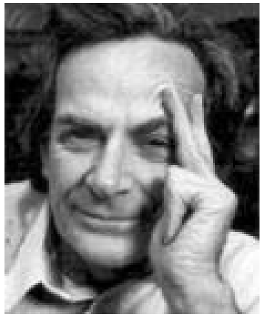
Characteristic of Quantum mechanics

No general consensus

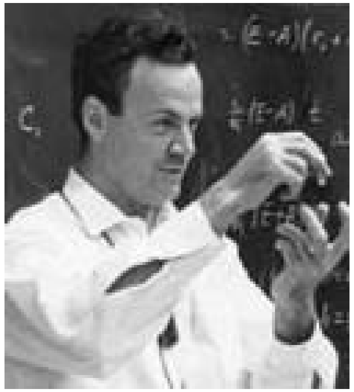
Can “do”, but can’t tell what we are doing.

Implausible

Niels Bohr: “If you are not confused by quantum physics then you haven’t really understood it”.



Richard Feynman: ”I think I can safely say that nobody understands quantum mechanics”.



Quantum Mechanics

- At the turn of the last century, there were several experimental observations which could not be explained by the established laws of classical physics and called for a radically different way of thinking

- This led to the development of Quantum Mechanics which is today regarded as the fundamental theory of Nature

Some key events/observations that led to the development of quantum mechanics…

- Black body radiation spectrum (Planck, 1901)
- Photoelectric effect (Einstein, 1905)
- Model of the atom (Rutherford, 1911)
- Quantum Theory of Spectra (Bohr, 1913)
- Scattering of photons on electrons (Compton, 1922)
- Exclusion Principle (Pauli, 1922)
- Matter Waves (de Broglie 1925)
- Experimental test of matter waves (Davisson and Germer, 1927)

Quantum Mechanics

- Matter and radiation have a dual nature – of both wave and particle

- The matter wave associated with a particle has a de Broglie wavelength given by

$$\boxed{\lambda = \frac{h}{p}}$$

- The wave corresponding to a quantum system is described by a wave function or state vector

$$|\psi\rangle$$

Introduction to Quantum mechanics

Schrödinger equation (1926)

$$i\hbar\frac{\partial\Psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2} + V\Psi$$

Erwin **Schrödinger**
1887-1961

Introduction to Quantum mechanics

Schrödinger equation (1926)

$$\left.\begin{array}{l}\boxed{i\hbar\frac{\partial\Psi}{\partial t}=H\Psi}\\ \\ \boxed{H=\frac{p^2}{2m}+V}\end{array}\right\}\quad i\hbar\frac{\partial\Psi}{\partial t}=-\frac{\hbar^2}{2m}\nabla^2\Psi+V\Psi$$

Introduction to Quantum mechanics

Schrödinger equation

$$\boxed{i\hbar\frac{\partial\Psi}{\partial t}=-\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2}+V\Psi}$$

m the mass of the particle

$\hbar$ the Planck's constant $\hbar=\frac{h}{2\pi}=1.05\times10^{-34}\,Js$

V the potential in which the particle exists

Ψ the "wave function" of the particle

But what is the physical meaning of the wave function?

Introduction to Quantum mechanics

Wave function

The wave function $\Psi(x,t)$ represents the “state of the particle”

Born’s Statistical interpretation

$\left|\Psi(x,t)\right|^2$ probability of finding the particle at point *x*, at time *t*

$\int_a^b \left|\Psi(x,t)\right|^2$ probability of finding the particle between points *a* and *b* at time *t*

Probabilities

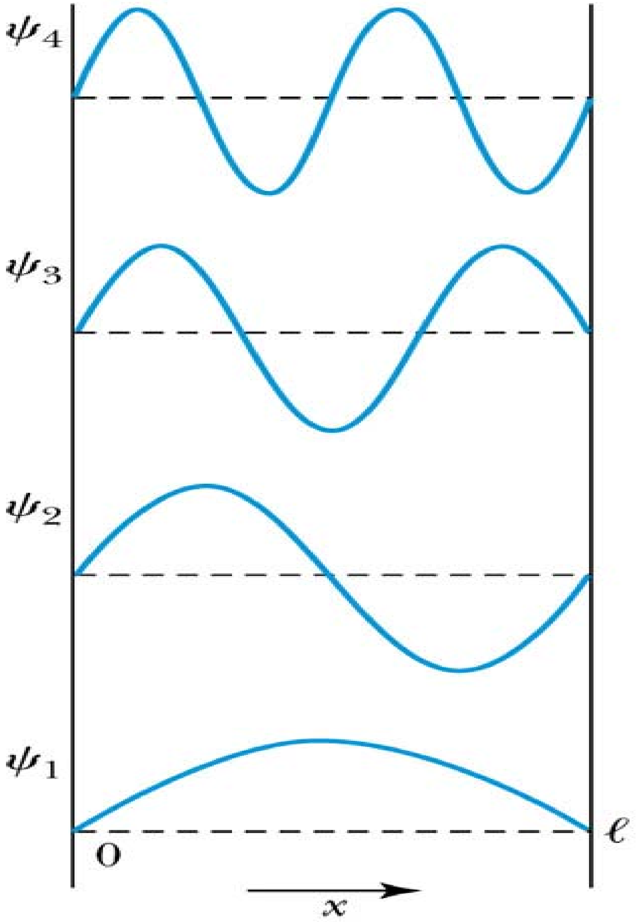
Often what we measure in an experiment is the probability density, $|\psi(x)|^2$.

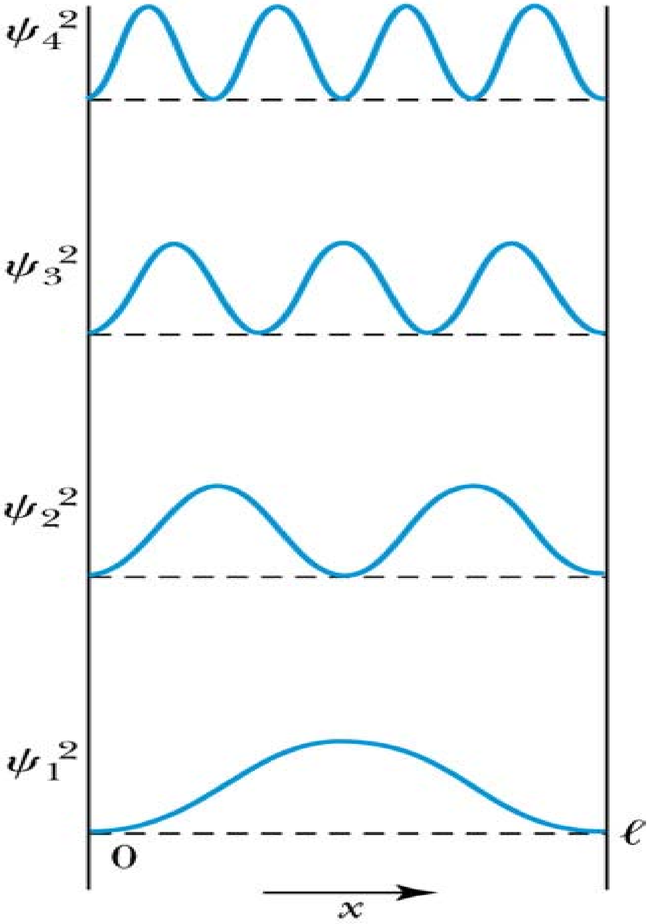
$$\psi_n(x) = N \sin\left(\frac{n\pi}{L}x\right)$$

Wave function = Probability amplitude

$$|\psi_n(x)|^2 = N^2 \sin^2\left(\frac{n\pi}{L}x\right)$$

Probability per unit length (in 1-dimension)





Introduction to Quantum mechanics

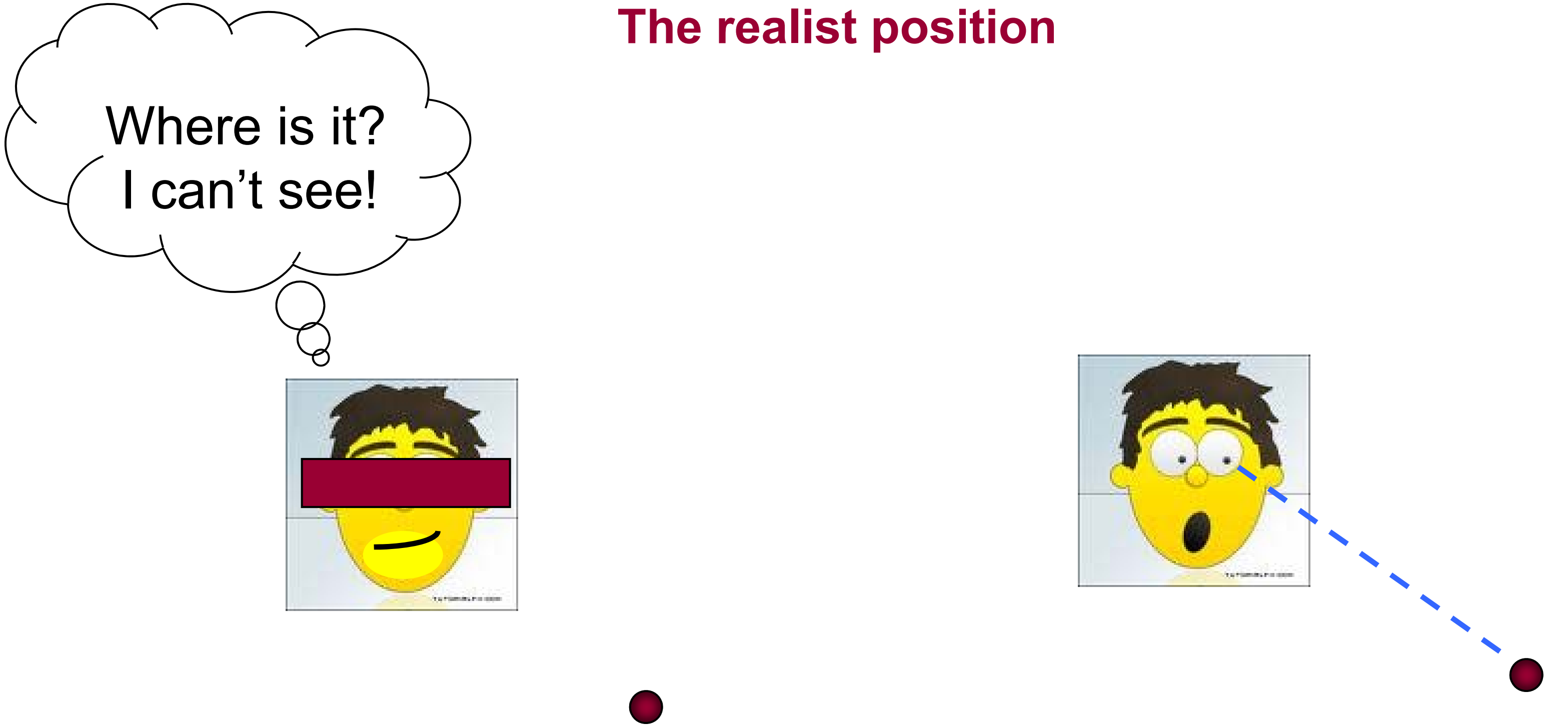
Indeterminacy

Quantum mechanics only offers a *statistical interpretation* about the *possible results* of a measurement

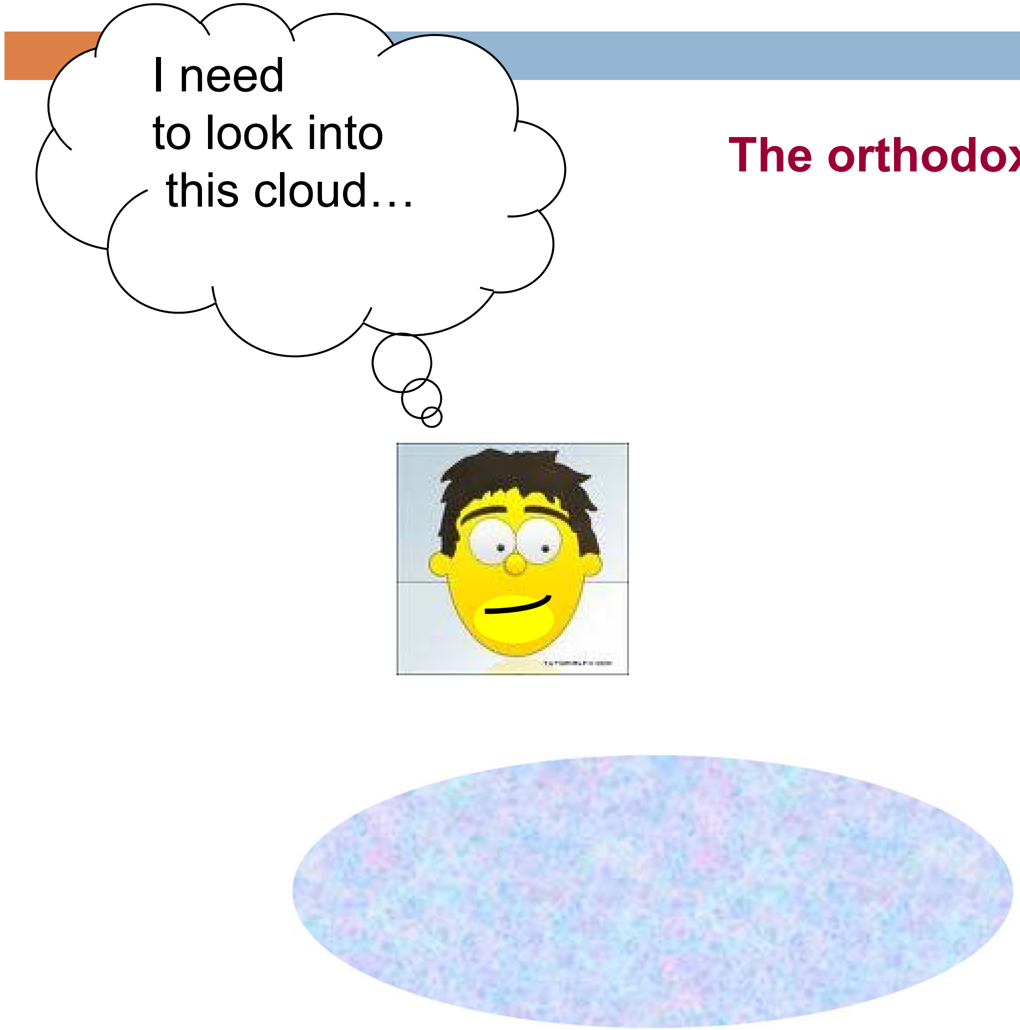
- Realist Position
- Orthodox position
- Agnostic position

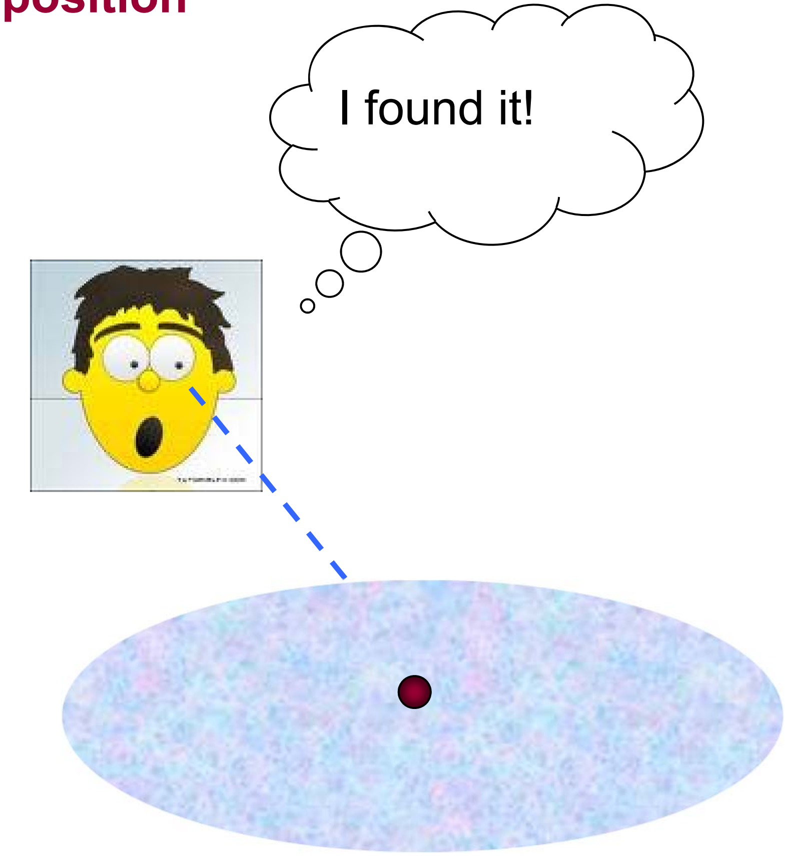
Introduction to Quantum mechanics

The realist position



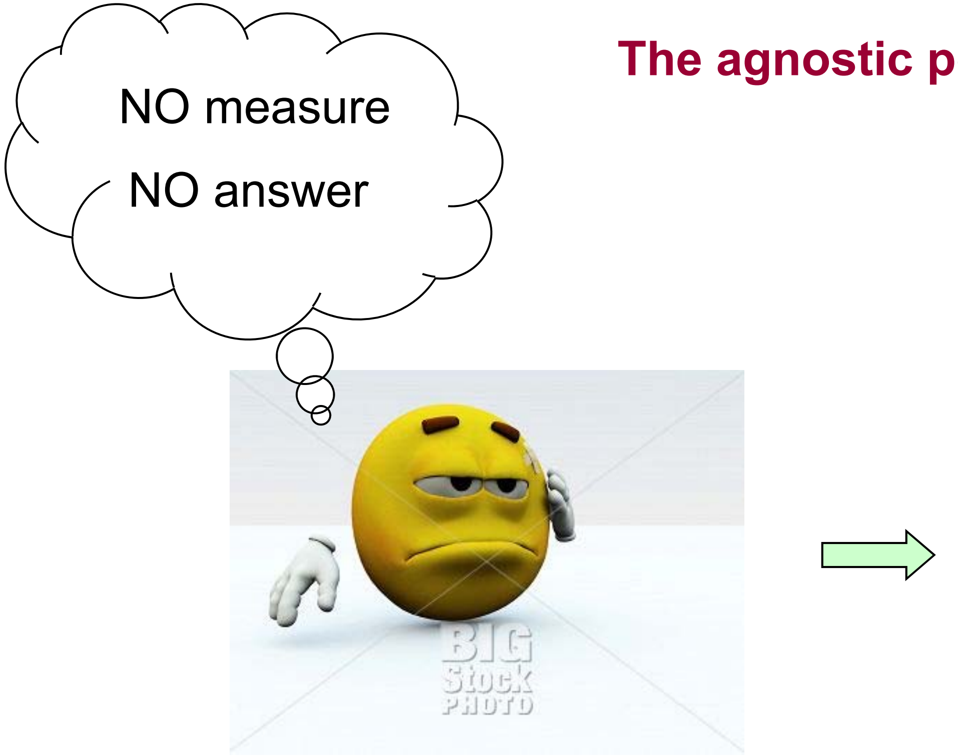
The orthodox position



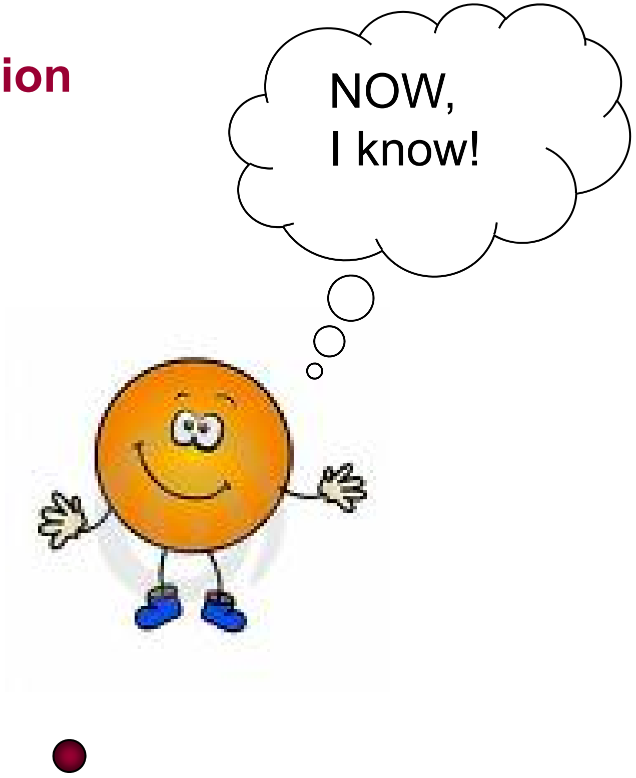


Introduction to Quantum mechanics

The agnostic position



“If I can’t see it, it doesn’t exist.”



“seeing is believing”

Connection to Wave function

Density of probability (now function of space and time):

$$\boxed{\rho(x,t) = \left|\Psi(x,t)\right|^2}$$

Normalization: $$\int_{-\infty}^{+\infty} \left|\Psi(x,t)\right|^2 = 1$$

Solutions $\Psi(x,t)$ have to be normalizable:

- needs to be square-integrable

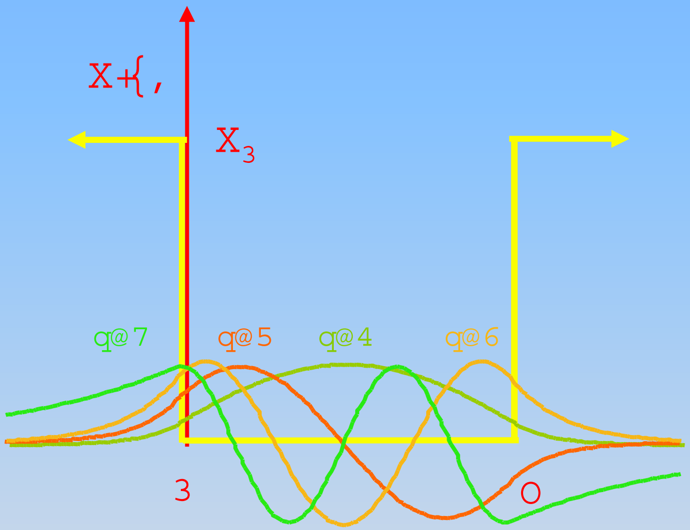
Normalization of Wave function

Normalization: $$\int_{-\infty}^{+\infty} \left|\Psi(x,t)\right|^2 = 1$$

Evolution of Ψ in time?

If Y satisfies the Schrödinger equation and is normalizable, then

$$\frac{d}{dt}\left(\int_{-\infty}^{+\infty} \left|\Psi(x,t)\right|^2\right) = 0$$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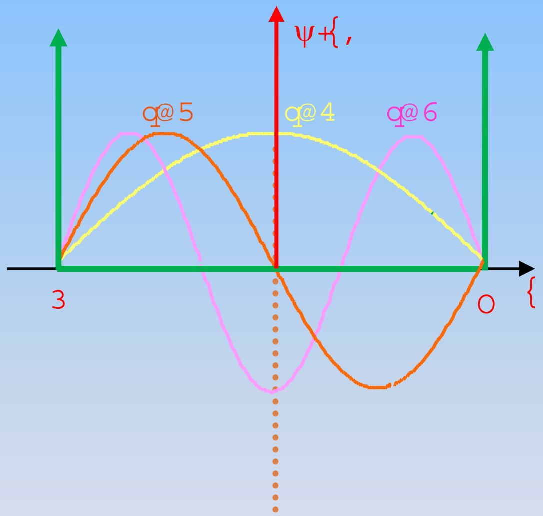
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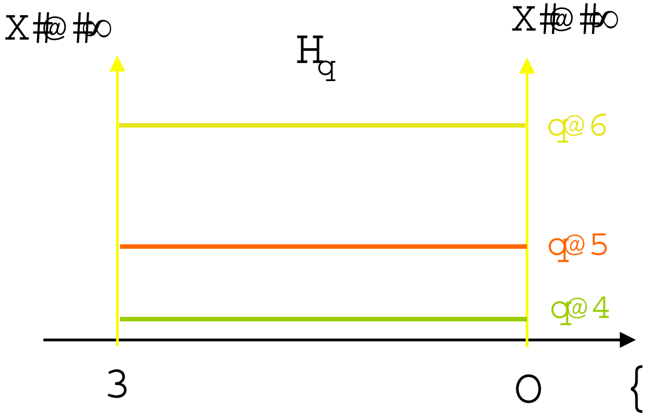
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